

Standard axioms for $|$ and **nil**:

$$P|\mathbf{nil} \equiv P \quad (P|Q)|R \equiv P|(Q|R) \quad P|Q \equiv Q|P \text{ if } P : 0$$

Standard scope axioms:

$$\begin{aligned} (\nu x)(P|Q) &\equiv (\nu x)P|Q && \text{if } x \notin fn(Q) && (\nu x)(\nu y)P \equiv (\nu y)(\nu x)P \\ (\nu x)(P|Q) &\equiv P|(\nu x)Q && \text{if } x \notin fn(P) \end{aligned}$$

Fusion axioms:

$$\begin{aligned} \langle x=x \rangle &\equiv \mathbf{nil} && \langle x=y \rangle | x.P \equiv \langle x=y \rangle | y.P && \langle x=y \rangle | \langle x \rangle \equiv \langle x=y \rangle | \langle y \rangle \\ (\nu x)\langle x=y \rangle &\equiv \mathbf{nil} && \langle x=y \rangle | \bar{x}.P \equiv \langle x=y \rangle | \bar{y}.P && \langle x=y \rangle | z.P \equiv \langle x=y \rangle | z.(\langle x=y \rangle | P) \\ \langle x=y \rangle &\equiv \langle y=x \rangle && \langle x=y \rangle | \langle x=z \rangle \equiv \langle x=y \rangle | \langle y=z \rangle && \langle x=y \rangle | \bar{z}.P \equiv \langle x=y \rangle | \bar{z}.(\langle x=y \rangle | P) \end{aligned}$$

The structural congruence between π_F -process, written \equiv , is the smallest equivalence relation satisfying these axioms and closed with respect to contexts

Can factor out datums and fusions to get *standard form*

$$\langle \vec{u}=\vec{v} \rangle \mid (\nu \vec{x})(\langle \vec{y} \rangle \mid P),$$

where \vec{x} s distinct and contained in the \vec{y} s, P contains no datums or fusions in its top level.

Interface $\langle \vec{u}=\vec{v} \rangle \mid (\nu \vec{x})(\langle \vec{y} \rangle \mid _)$ is unique: given

$$\langle \vec{u}_1=\vec{v}_1 \rangle \mid (\nu \vec{x}_1)(\langle \vec{y}_1 \rangle \mid P_1) \equiv \langle \vec{u}_2=\vec{v}_2 \rangle \mid (\nu \vec{x}_2)(\langle \vec{y}_2 \rangle \mid P_2),$$

the fusions $\langle \vec{u}_1=\vec{v}_1 \rangle$ and $\langle \vec{u}_2=\vec{v}_2 \rangle$ denote same equivalence relation $|\vec{x}_1| = |\vec{x}_2|$, and the datums \vec{y}_1, \vec{y}_2 are identical and the processes P_1, P_2 structurally congruent up to the name-equivalence and α -conversion of the \vec{x} s.

Connection operator \textcircled{C} connects interfaces. If P and Q have standard forms $\langle \vec{u}_1 = \vec{v}_1 \rangle | (\nu \vec{x}_1) (\langle \vec{y}_1 \rangle | P_1)$ and $\langle \vec{u}_2 = \vec{v}_2 \rangle | (\nu \vec{x}_2) (\langle \vec{y}_2 \rangle | P_2)$ respectively, then

$$P \textcircled{C} Q \stackrel{\text{def}}{=} \langle \vec{u}_1 \vec{u}_2 = \vec{v}_1 \vec{v}_2 \rangle | (\nu \vec{x}_1 \vec{x}_2) (\langle \vec{y}_1 = \vec{y}_2 \rangle | P_1 | P_2),$$

Fusion calculus processes:

$$P ::= \mathbf{nil} \mid P|P \mid (\nu x)P \mid z\vec{x}.P \mid \bar{z}\vec{x}.P.$$

Fusion calculus reaction relation

$$(\nu \vec{u})(z\vec{x}.P \mid \bar{z}\vec{y}.Q \mid R) \searrow P\sigma \mid Q\sigma \mid R\sigma,$$

where $\text{ran}(\sigma), \text{dom}(\sigma) \subseteq \{\vec{x}, \vec{y}\}$ and $\vec{u} = \text{dom}(\sigma) \setminus \text{ran}(\sigma)$ and $\sigma(v) = \sigma(w)$ if and only if $(v, w) \in E(\langle \vec{x} = \vec{y} \rangle)$.

To explain side-condition: Consider the equivalence relation generated from the equalities $\vec{x} = \vec{y}$. The side-conditions ensure that, for each equivalence class, every element is mapped by σ to a single free witness.

Or, equivalently, “there may be no free fusions left”

$$\begin{array}{c}
x.P \xrightarrow{x} P \quad \bar{x}.P \xrightarrow{\bar{x}} P \\
x.P \mid \bar{y}.Q \xrightarrow{?x=y} P \circledast Q \quad x.P \mid \bar{x}.Q \xrightarrow{\tau} P \circledast Q \\
\frac{P \xrightarrow{\alpha} P'}{P \mid Q \xrightarrow{\alpha} P' \mid Q} \quad \frac{P \xrightarrow{\alpha} P'}{Q \mid P \xrightarrow{\alpha} Q \mid P'} \quad \frac{P \xrightarrow{\alpha} P', \quad x \notin \alpha}{(\nu x)P \xrightarrow{\alpha} (\nu x)Q} \quad \frac{P' \equiv P \xrightarrow{\alpha} Q \equiv Q'}{P' \xrightarrow{\alpha} Q'}
\end{array}$$

Quotiented labelled transition system. We do not distinguish between $?x=y$ and $?y=x$. The final rule closes the LTS with respect to the structural congruence.

Fusion bisimulation. A symmetric relation \mathcal{S} is a *fusion bisimulation* iff whenever $P\mathcal{S}Q$ then

1. $P, Q : m > 0$ implies P and Q have standard forms $\langle \vec{u}=\vec{v} \rangle | (\nu \vec{x}) (\langle \vec{y} \rangle | P_1)$ and $\langle \vec{u}=\vec{v} \rangle | (\nu \vec{x}) (\langle \vec{y} \rangle | Q_1)$ respectively and $\langle \vec{u}=\vec{v} \rangle | P_1 \mathcal{S} \langle \vec{u}=\vec{v} \rangle | Q_1$;
2. $P, Q : 0$ implies they have standard forms $\langle \vec{u}=\vec{v} \rangle | P_1$ and $\langle \vec{u}=\vec{v} \rangle | Q_1$, and
 - (a) if $P \xrightarrow{\alpha} P'$ where α is x, \bar{x} or τ , then $Q \xrightarrow{\alpha} Q'$ and $P'\mathcal{S}Q'$
 - (b) if $P \xrightarrow{?x=y} P'$ then either $Q \xrightarrow{?x=y} Q'$ or $Q \xrightarrow{\tau} Q'$, and $\langle x=y \rangle | P'\mathcal{S} \langle x=y \rangle | Q'$;
3. similarly for Q .

$$\begin{array}{c}
x.P \xrightarrow{x}_s P \qquad \bar{x}.P \xrightarrow{\bar{x}}_s P \qquad \frac{P \xrightarrow{\alpha_1}_s Q \quad \alpha_1 =_{E(P)} \alpha_2}{P \xrightarrow{\alpha_2}_s Q} * \\
\\
\frac{P \xrightarrow{x}_s P' \quad Q \xrightarrow{\bar{y}}_s Q'}{P|Q \xrightarrow{?x=y}_s P'@Q'} \quad \frac{P \xrightarrow{\bar{x}}_s P' \quad Q \xrightarrow{y}_s Q'}{P|Q \xrightarrow{?x=y}_s P'@Q'} \quad \frac{P \xrightarrow{?x=x}_s Q}{P \xrightarrow{\tau}_s Q} \\
\\
\frac{P \xrightarrow{\alpha}_s P'}{P|Q \xrightarrow{\alpha}_s P'|Q} \quad \frac{P \xrightarrow{\alpha}_s P'}{Q|P \xrightarrow{\alpha}_s Q|P'} \quad \frac{P \xrightarrow{\alpha}_s Q, \quad x \notin \alpha}{(\nu x)P \xrightarrow{\alpha}_s (\nu x)Q}
\end{array}$$

* We write $\alpha =_{E(P)} \beta$ if α, β are identical up to $E(P)$

Structured labelled transition system. This LTS does not include a rule involving the structural congruence. We write $E(P)$ for the equivalence relation on names generated by P . A simple characterisation is given by $(x, y) \in E(P)$ if and only if $P \equiv P|\langle x=y \rangle$.

Comparison of labelled transitions: fusion calculus, π_F -calculus.

(fusion calculus)	(π_F -calculus)
$\bar{u}x.P \xrightarrow{fu} P$	$\bar{u}.(\langle x \rangle P^*) \xrightarrow{\pi_F} \langle x \rangle P^*$
$(\nu x)\bar{u}x.P \xrightarrow{fu} P$	$(\nu x)\bar{u}.(\langle x \rangle P^*) \xrightarrow{\pi_F} (\nu x)(\langle x \rangle P^*)$
$\bar{u}x.P uy.Q \xrightarrow{fu} P Q$	$\bar{u}.(\langle x \rangle P^*) u.(\langle y \rangle Q^*) \xrightarrow{\pi_F} \langle x=y \rangle P^* Q^*$

The fusion label $x=y$ states that a fusion has occurred as a consequence of a reaction. Not the same as the label $?x=y$ in the π_F -calculus, which states that an external fusion must be present for reaction to occur.

The additional information conveyed by a fusion calculus label, is conveyed in the π_F -calculus by the interface of the resulting process.

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