Standard axioms for $|$ and **nil**:

$$P|\textbf{nil} \equiv P \qquad (P|Q)|R \equiv P|(Q|R) \qquad P|Q \equiv Q|P \text{ if } P : 0$$

Standard scope axioms:

$$(\nu x)(P|Q) \equiv (\nu x)P|Q \quad \text{if } x \notin fn(Q) \qquad (\nu x)(\nu y)P \equiv (\nu y)(\nu x)P$$
$$(\nu x)(P|Q) \equiv P|(\nu x)Q \quad \text{if } x \notin fn(P)$$

Fusion axioms:

$$\langle x{=}x\rangle \equiv \textbf{nil} \qquad \langle x{=}y\rangle \mid x.P \equiv \langle x{=}y\rangle \mid y.P \qquad \langle x{=}y\rangle \mid \langle x\rangle \equiv \langle x{=}y\rangle \mid \langle y\rangle$$
$$(\nu x)\langle x{=}y\rangle \equiv \textbf{nil} \qquad \langle x{=}y\rangle \mid \overline{x}.P \equiv \langle x{=}y\rangle \mid \overline{y}.P \qquad \langle x{=}y\rangle \mid z.P \equiv \langle x{=}y\rangle \mid z.(\langle x{=}y\rangle|P)$$
$$\langle x{=}y\rangle \equiv \langle y{=}x\rangle \qquad \langle x{=}y\rangle \mid \langle x{=}z\rangle \equiv \langle x{=}y\rangle \mid \langle y{=}z\rangle \qquad \langle x{=}y\rangle \mid \overline{z}.P \equiv \langle x{=}y\rangle \mid \overline{z}.(\langle x{=}y\rangle|P)$$

The structural congruence between π_F-process, written $\equiv$, is the smallest equivalence relation satisfying these axioms and closed with respect to contexts

Can factor out datums and fusions to get *standard form*

$$\langle\vec{u}{=}\vec{v}\rangle \mid (\nu\vec{x})(\langle\vec{y}\rangle \mid P),$$

where $\vec{x}$s distinct and contained in the $\vec{y}$s, P contains no datums or fusions in its top level.

Interface $\langle\vec{u}{=}\vec{v}\rangle|(\nu\vec{x})(\langle\vec{y}\rangle|\ \_\ )$ is unique: given

$$\langle\vec{u}_1{=}\vec{v}_1\rangle \mid (\nu\vec{x}_1)(\langle\vec{y}_1\rangle|P_1) \;\equiv\; \langle\vec{u}_2{=}\vec{v}_2\rangle \mid (\nu\vec{x}_2)(\langle\vec{y}_2\rangle|P_2),$$

the fusions $\langle\vec{u}_1{=}\vec{v}_1\rangle$ and $\langle\vec{u}_2{=}\vec{v}_2\rangle$ denote same equivalence relation $|\vec{x}_1| = |\vec{x}_2|$, and the datums $\vec{y}_1, \vec{y}_2$ are identical and the processes P_1, P_2 structurally congruent up to the name-equivalence and α-conversion of the $\vec{x}$s.

Connection operator $@$ connects interfaces. If P and Q have standard forms $\langle\vec{u}_1{=}\vec{v}_1\rangle|(\nu\vec{x}_1)(\langle\vec{y}_1\rangle|P_1)$ and $\langle\vec{u}_2{=}\vec{v}_2\rangle|(\nu\vec{x}_2)(\langle\vec{y}_2\rangle|P_2)$ respectively, then

$$P@Q \stackrel{\text{def}}{=} \langle\vec{u}_1\vec{u}_2{=}\vec{v}_1\vec{v}_2\rangle \mid (\nu\vec{x}_1\vec{x}_2)(\langle\vec{y}_1{=}\vec{y}_2\rangle|P_1|P_2),$$

Fusion calculus processes:

$$P ::= \textbf{nil} \;\Big|\; P|P \;\Big|\; (\nu x)P \;\Big|\; z\vec{x}.P \;\Big|\; \overline{z}\vec{x}.P.$$

Fusion calculus reaction relation

$$(\nu\vec{u})(z\vec{x}.P \mid \overline{z}\vec{y}.Q \mid R) \;\searrow\; P\sigma \mid Q\sigma \mid R\sigma,$$

where $\mathrm{ran}(\sigma), \mathrm{dom}(\sigma) \subseteq \{\vec{x}, \vec{y}\}$ and $\vec{u} = \mathrm{dom}(\sigma) \backslash \mathrm{ran}(\sigma)$ and $\sigma(v) = \sigma(w)$ if and only if $(v, w) \in E(\langle\vec{x}{=}\vec{y}\rangle)$.

To explain side-condition: Consider the equivalence relation generated from the equalities $\vec{x}{=}\vec{y}$. The side-conditions ensure that, for each equivalence class, every element is mapped by σ to a single free witness.

Or, equivalently, “there may be no free fusions left”

$$x.P \xrightarrow{x} P \qquad \overline{x}.P \xrightarrow{\overline{x}} P$$

$$x.P \mid \overline{y}.Q \xrightarrow{?x=y} P@Q \qquad x.P \mid \overline{x}.Q \xrightarrow{\tau} P@Q$$

$$\frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q} \quad \frac{P \xrightarrow{\alpha} P'}{Q|P \xrightarrow{\alpha} Q|P'} \quad \frac{P \xrightarrow{\alpha} P', \quad x \notin \alpha}{(\nu x)P \xrightarrow{\alpha} (\nu x)Q} \quad \frac{P' \equiv P \xrightarrow{\alpha} Q \equiv Q'}{P' \xrightarrow{\alpha} Q'}$$

Quotiented labelled transition system. We do not distinguish between $?x{=}y$ and $?y{=}x$. The final rule closes the LTS with respect to the structural congruence.

Fusion bisimulation. A symmetric relation $\mathcal{S}$ is a *fusion bisimulation* iff whenever $P\mathcal{S}Q$ then

1. $P, Q : m > 0$ implies P and Q have standard forms $\langle\vec{u}{=}\vec{v}\rangle|(\nu\vec{x})(\langle\vec{y}\rangle|P_1)$ and $\langle\vec{u}{=}\vec{v}\rangle|(\nu\vec{x})(\langle\vec{y}\rangle|Q_1)$ respectively and $\langle\vec{u}{=}\vec{v}\rangle|P_1\ \mathcal{S}\ \langle\vec{u}{=}\vec{v}\rangle|Q_1$;

2. P, Q : 0 implies they have standard forms $\langle\vec{u}{=}\vec{v}\rangle|P_1$ and $\langle\vec{u}{=}\vec{v}\rangle|Q_1$, and

   (a) if $P \xrightarrow{\alpha} P'$ where α is x, $\overline{x}$ or τ, then $Q \xrightarrow{\alpha} Q'$ and $P'\mathcal{S}Q'$

   (b) if $P \xrightarrow{?x=y} P'$ then either $Q \xrightarrow{?x=y} Q'$ or $Q \xrightarrow{\tau} Q'$, and $\langle x{=}y\rangle|P'\mathcal{S}\langle x{=}y\rangle|Q'$;

3. similarly for Q.

$$x.P \xrightarrow{x}_s P \qquad \overline{x}.P \xrightarrow{\overline{x}}_s P \qquad \frac{P \xrightarrow{\alpha_1}_s Q \quad \alpha_1 =_{E(P)} \alpha_2}{P \xrightarrow{\alpha_2}_s Q}*$$

$$\frac{P \xrightarrow{x}_s P' \quad Q \xrightarrow{\overline{y}}_s Q'}{P|Q \xrightarrow{?x=y}_s P'@Q'} \qquad \frac{P \xrightarrow{\overline{x}}_s P' \quad Q \xrightarrow{y}_s Q'}{P|Q \xrightarrow{?x=y}_s P'@Q'} \qquad \frac{P \xrightarrow{?x=x}_s Q}{P \xrightarrow{\tau}_s Q}$$

$$\frac{P \xrightarrow{\alpha}_s P'}{P|Q \xrightarrow{\alpha}_s P'|Q} \qquad \frac{P \xrightarrow{\alpha}_s P'}{Q|P \xrightarrow{\alpha}_s Q|P'} \qquad \frac{P \xrightarrow{\alpha}_s Q, \quad x \notin \alpha}{(\nu x)P \xrightarrow{\alpha}_s (\nu x)Q}$$

* We write $\alpha =_{E(P)} \beta$ if α, β are identical up to $E(P)$

Structured labelled transition system. This LTS does not include a rule involving the structural congruence. We write $E(P)$ for the equivalence relation on names generated by P. A simple characterisation is given by $(x, y) \in E(P)$ if and only if $P \equiv P|\langle x{=}y\rangle$.

Comparison of labelled transitions: fusion calculus, π_F-calculus.

(fusion calculus)			(π_F-calculus)		
$\overline{u}x.P$	$\xrightarrow{\overline{u}x}_{fu}$	P	$\overline{u}.(\langle x\rangle \vert P^*)$	$\xrightarrow{\overline{u}}_{\pi_F}$	$\langle x\rangle \vert P^*$
$(\nu x)\overline{u}x.P$	$\xrightarrow{(x)\overline{u}x}_{fu}$	P	$(\nu x)\overline{u}.(\langle x\rangle \vert P^*)$	$\xrightarrow{\overline{u}}_{\pi_F}$	$(\nu x)(\langle x\rangle \vert P^*)$
$\overline{u}x.P \mid uy.Q$	$\xrightarrow{x=y}_{fu}$	$P \mid Q$	$\overline{u}.(\langle x\rangle \vert P^*) \mid u.(\langle y\rangle \vert Q^*)$	$\xrightarrow{\tau}_{\pi_F}$	$\langle x{=}y\rangle \mid P^* \mid Q^*$

The fusion label $x{=}y$ states that a fusion has occurred as a consequence of a reaction. Not the same as the label ?$x{=}y$ in the π_F-calculus, which states that an external fusion must be present for reaction to occur.

The additional information conveyed by a fusion calculus label, is conveyed in the π_F-calculus by the interface of the resulting process.

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